Robert Fäßler, 3th of February 2022 → this abstract is based on of my thoughts, drafts and presentations starting in December 2013 for my doctoral thesis

- 25th of July 2022 - extension for production technology and a proof for the existence of a Recursive Competitive Equilibrium with a pareto optimum

Asset Pricing and exhaustible Resources

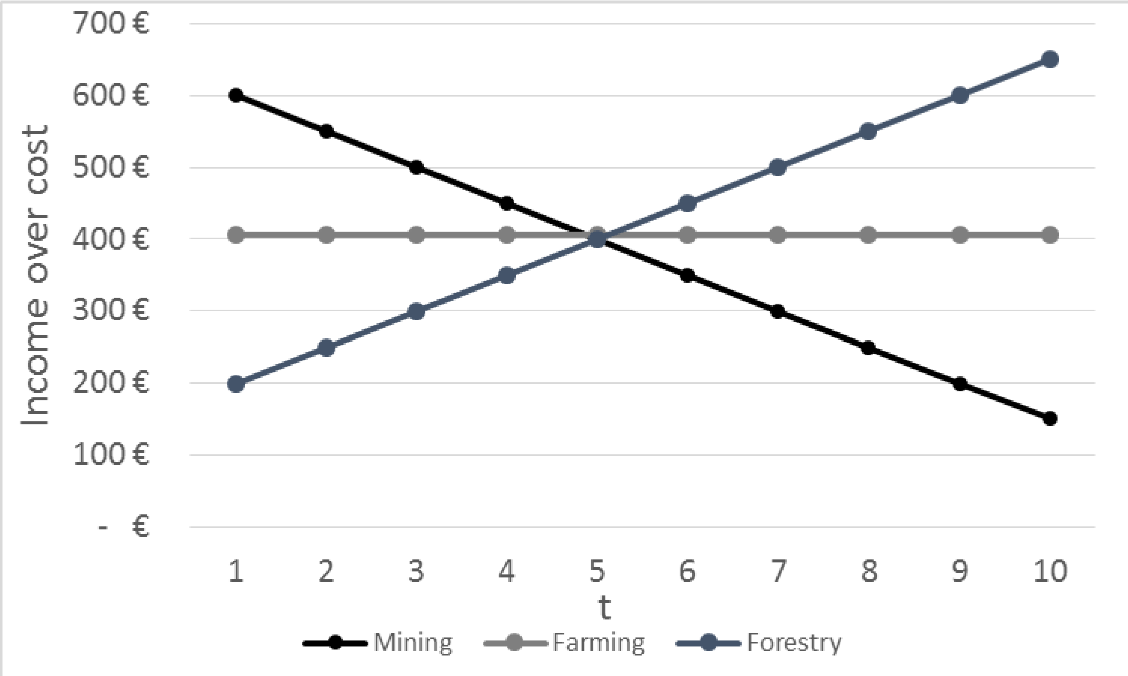
1. Introduction

In the following we define an economy which is an extension of the Lucas (1978)[1] type of economy, with an entirely exogenous production and where it is explicitly stated, that "no resources are utilized, and there is no possibility of affecting the output of any unit at any time". The extension is chosen for the reason that in our economy exist resources, which have a special characteristic which allow the economy as a whole, but not a single consumer or producer, to affect the output of units over time. This type of resources are commonly described as exhaustible resources and perfect consumption or extraction paths are described in the abstract of Harold Hotelling (1931)[2] "The Economics of Exhaustible Resources" where the rate of return in its classical interpretation is exogenously given. Such types of exhaustible resources are already described in the abstract of Irving Fisher (1930)[3] "the Theory of Interest, as determined by impatience to spend income and the opportunity to invest it". However, Fisher (1930) already referred to such types of exhaustible resources in the form of coal or ore mines as well as oil and gas sources, which physically determine the production opportunities in an economy over a time path and therefore are an important part of the classical rate of interest, which of course does not reflect risk in the form of modern asset pricing[4][5][6][7]. Let's for an example shortly compare two isolated economies, one which has a single factory and a mine which has a large stock which allows it to produce the same output as the factory with less afford and a second economy which solely has the factory.

The first economy is of course richer than the second economy and therefore the interest rate will be higher, if investors can generate higher returns by investing in the mine than investing in the factory or by combinations of their investments. However, we will see that such resources may just have a special characteristic which makes them reasonable for asset pricing and will have a closer look at this.

[1] Lucas, R. (1978): "Asset Prices in an Exchange Economy," Econometrica, 46, 1429-1445
[2] Hotelling, H. (1931): "The Economics of Exhaustible Resources", Journal of Political Economy 39, 137–175.
[3] Fisher, I. (1930): "The theory of interest, as determined by impatience to spend income and opportunity to invest it" Macmillan, New York 1930
[4] Tobin, J. (1971): "Liquidity Preference as Behavior Towards Risk", The Review of Economic Studies Vol. 25, No. 2 (Feb., 1958), pp. 65-86
[5] Ross, S: "The Arbitrage Theory of Capital Asset Pricing". In: Journal of Economic Theory. 1976, S. 341–360
[6] Zimmermann, H. (1998): "State-Preference Theorie und Asset Pricing", Studies in Contemporary Economics
[7] Arrow, K. J.: Aspects of the Theory of Risk Bearing. Helsinki: Yrjo Jahnsson, Lectures, 1965

A simplified example for different real investment opportunities and their time schedule with regard to the real market rate of interest is scheduled in the following and follows the logic of Fisher (1930):



t	Mining	Farming	Forestry
0	- 1.000 €	- 1.000 €	- 1.000 €
1	600 €	405 €	200 €
2	550 €	405 €	250 €
3	500 €	405 €	300 €
4	450 €	405 €	350 €
5	400 €	405 €	400 €
6	350 €	405 €	450 €
7	300 €	405 €	500 €
8	250 €	405 €	550 €
9	200 €	405 €	600 €
10	150 €	405 €	650 €
Income over cost	2.750 €	3.050 €	3.250 €
IRR	50%	39%	30%

Real Investment	Mining	Farming	Forestry
NPV (i=1%)	2.591 €	2.836 €	2.986 €
NPV (i=5%)	2.050 €	2.127 €	2.127 €
NPV (i=10%)	1.542 €	1.489 €	1.373 €

Figure 1: different time schemes of real investments
Source: Own example based on Fisher (1930)

The investment schedules lay out a general topic of investment calculations, where the internal rate of return of investments which generate a fast return is higher as for investments which generate a growing or constant return structure, even so the last ones have even higher overall returns. It is noteworthy that exhaustible resource extraction is strongly associated with investments with a fast return and therefore with high internal rate of returns. From this point of view it could be imagined, that the exhaustition or depletion of natural resources like oil, coal or metals could lead to decreasing interest rates over time, simply by the fact that high internal rate of return investment opportunities are depleted:

"If we look ahead to a distant time when all the resources of the earth will be near exhaustion, and the human race reduced to complete poverty, we may expect very high interest rates indeed." Harold Hotelling (1931): The Economics of Exhaustible Resources, In: Journal of Political Economy 39 (2), p. 145.

"If we live in a land covered with young forests or otherwise affording plenty of opportunities for distant income but affording few opportunities for immediate income the rate of interest will, other things being equal, be very much higher than in a land full of nearly worked out mines and oil fields or otherwise affording many opportunities for immediate but few opportunities for remote income." Irving Fisher (1930), The Theory of Interest, Part II, §4, p. 143.

The subject of real investment opportunities will be discussed in relation with the production technology under point 4 in more detail.

The topic of exhaustible resources, their sustainable use and their optimal extraction path have a special role in economics and are analyzed in many general equilibrium and optimization models, so in Solow (1974)[8], Stiglitz (1974)[9], Gaitan, Tol, Yetkiner (2006)[10] or Sinn (2008)[11] and have with regard to climate change still a high relevance.

[8] Solow, R.M. (1974): "The Economics of Resources or the Resources of Economics", American Economic Review 64, 1–14

[9] Stiglitz, J.E. (1974): "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths", Review of Economic Studies 41, Symposium on the Economics of Exhaustible Resources, 123–137

[10] Gaitan, B. Tol, R. S. I., Yetkiner, H. (2006): "The Hotelling's Rule Revisited in a Dynamic General Equilibrium Model." In: O. Esen, A. Ogus: Proceedings of the International Conference on Human and Economic Resources. Izmir: Izmir University of Economics

[11] Sinn, H. (2008): Das grüne Paradoxon: Warum man das Angebot bei der Klimapolitik nicht vergessen darf, Vol. 9 (Special Issue), S. 125–126

2. About the nature of the special value of exhaustible resources and why it is important for asset pricing

We will define exhaustible resources in our economy in the following by a type of resource, with the following characteristics:

I. Exhaustible resources are given exogenously by nature and resources like oil or gas are explored in a more or less random pattern on landscapes.

II. “” are non reproducible for the reason that they are given by nature but to be more precise for the reason, that they can not be reproduced in the economy's production processes. Let us also be precise about this assumption and apply it to our existing economy, where of course there are a lot of substitutes to e.g. a unit of oil, due to the fact that the consumption value of a unit is mainly given by its energy value. But as long as there exists no alternative production opportunity in our economy, which allows to reproduce such a unit of energy to the same or a lower price, this unit has a special exhaustible resource value.

III. “ allow their owners, the economy as a whole and therefore our representative household the opportunity for an additional consumption stream. However, following assumptions I. and II. let us be precise about what makes this consumption stream special. Therefore let us first look at the microeconomic perspective of a single household who owns for example an oil source. As the endowment of exhaustible sources is given by nature and their endowment is random about the space of the earth there are two possible cases:
a) the household has either acquired this oil source or the piece of land with the knowledge of the existence of the oil source and therefore he has paid a price which already reflects the additional consumption stream. As we assume the general framework of a Lucas (1978) economy with a stand in household we can be sure that the representative household has paid a fair price for this oil source, which reflects the discounted expected consumption streams. Therefore this oil source does not generate a consumption stream, which affords a special recognition for this analysis.
b) If exhaustible resources are randomly distributed above our earth and were at least initially explored by accident, then there are households, countries, communities or other types of organizations who found that they are the lucky owners of a valuable exhaustible resource. Lets for example again take the oil source found on the farming field, for which the owner has not paid a price which is equal to the value of its expected consumption streams. The other way around, some exhaustible resources like for example uran or in former times oil sources were more or less not very useful or even had a negative value until the technology has been invented to utilize them. Again the owner nowadays has an additional value which leads to an additional consumption stream. The stand in household is representative for the whole economy and therefore he or she holds such an exhaustible resource with a special consumption value.

IV. If I, II and III hold, a representative household holds an exhaustible resource stock which reflects a special value which was given by chance (fortunie) and for the economy as a whole by nature. Always looking at the microeconomic perspective, how will a household use this special value if we take into account that for reasons of marginal rate of substitution between goods he will not have the highest utility by consuming e.g. an oil stock alone. Here we have a microeconomic optimization problem for the household due to the fact that e.g. the oil source has to be transformed into consumption by means of production or more simply in an exchange economy by sales to productive units. Now before we look at the result of the equilibrium with a representative consumer let us look at the way how such special values of exhaustible resources are distributed in reality. For e.g. oil sources there are some larger entities, usually governments and even organizations like OPEC, who hold this special value. But in fact they deal with oil and not the special value of the resource oil so they have to be competitive to the closest competitive oil producer or substitute to oil. And in such a way all exhaustible resources with a special value, which are many and not only oil e.g. metals have a special value which is the difference between extraction costs and e.g. recycling or alternative metal extraction costs, have some kind of market structure which is in fact not monopolistic. However, for most of the exhaustible natural resources the stocks are in some way concentrated but we could imagine other exhaustible resources for example the most straightforward is lifetime itself which are not concentrated. This leads in some kind to the nature of optimization problems for exhaustible resources, as their optimal consumption or in the first step extraction path, which contains in reality some kind of extraction costs and investments for extraction, is to find an optimal time path. For an optimal time path of extraction for every household the consumption value has to be maximized and investment schedules with respect to risk have to be planned, which is in fact the same optimization problem as in the Lucas (1978)[12] economy but with a special consumption value of exhaustible resources. For a household who finds an oil source on its farming field we would assume that the best solution, especially under circumstances of risk diversification, is to sell to a specialized extraction company which is a part of the market portfolio or in oil-producing countries to the government or its institutions.

But then the household has a wealth stock increase which was realized like a lottery win at this point in time and therefore he has to adjust its saving and consumption time patterns as he can not or with only very low probability expect such an additional income with a special exhaustible resource value again. Note that this is a matter of time path of consumption and not a matter of behavior toward risk, especially not if we assume a utility function of consumption of CRRA type. On the other hand the extraction company or government institution acquiring the oil source still has to find an optimal extraction path over time as immediately extracting all would afford e.g. higher initial investments as slowly extracting. Of course we could think that the seller now lends more money to the market, which the extraction company borrows. In any case, we have to find an optimal extraction path which leads to an optimal

[12] Lucas, R. (1978): "Asset Prices in an Exchange Economy," Econometrica, 46, 1429-1445

consumption path for the special resource value before the transaction, to generate a fair valuation for both parties.

Let us have a look at the optimal extraction path for the exhaustible resource subject to the Hotelling (1931)[13] rule under conditions of free competition where the resource owner maximizes its utility of resource use and where no other complicating factors exist. The resource owner has to decide if he receives for a unit of its product q a price P_0 today or a price P_t in period t. Let us be precise P denotes according to Hotelling (1931)[14] exactly the net price received after paying the cost of extraction (for e.g. a mine or oil source owner). We should be cautious here because P should be equal to 0 after paying e.g. labor and capital costs, if the exhaustible resource would not have the special value subject to I, II, III and IV. Now let us maximize the utility of the extraction of q according to Hotelling (1931):

(1) $u(q) = \int_0^q p(q)dq,$

where the integrand is a diminishing function and the upper limit is the quantity actually placed upon the market and consumed. If future enjoyment is discounted by the force of interest r, the special present value of the exhaustible resource stock is:

(2) $V = \int_0^T u[q(t)]e^{-rt}dt.$

Let us for the moment define r as the rate of "pure" impatience, as in a classical interpretation market rate of interest refers to impatience, a utility function which evaluates future and present consumption and the expected level of future and present consumption. It is worth to mention that r and P are given by the market to the extractor under free competition and in Fama (1970)[15] terms, that prices as well as interest rates "fully reflect all available information". Now let's follow Hotelling again, "since $\int_0^T qdt$ is fixed, the production schedule $q(t)$ which makes V a maximum must be such that a unit increment in q will increase the integrand as much at one time as at another. That is,

(3) $\frac{d}{dq}u[q(t)]e^{-rt},$

which by (1) equals Pe^{-rt}, is a constant. Calling this constant P_0 we have:

(4) $P = P_0e^{rt},$

as result under conditions of free competition." Note that this path is likewise the maximizing social value as well as the optimal result under free competition.

[13] Hotelling, H. (1931): "The Economics of Exhaustible Resources", Journal of Political Economy 39, 137–175

[14] Hotelling, H. (1931): "The Economics of Exhaustible Resources", Journal of Political Economy 39, 137–175

[15] Fama, E. F.: "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, 25 (1970), 387-417

Now where is the problem with this if we look at a Lucas (1978) economy or if we want to use this in a consumption based asset pricing model. The problem is, that utility only arises from consumption and therefore a utility function should reflect consumption subject to q and not q. As mentioned above in an exchange economy it is straightforward to transfer product q to consumption by exchanging it against the single consumption good c. Then it holds that consumption related to q which we will define by Cq is equal to:

(5) $Cq_t = P_t * q_t$.

For the reason that the resource owner is a price taker we can rewrite the intertemporal optimization problem of extraction in an intertemporal optimization of consumption subject to the special exhaustible resource value:

(6) $V = \int_0^T u[p(t) * q(t)]e^{-rt}dt = \int_0^T u[Cq(t)]e^{-rt}dt$.

Note that this is the same intertemporal optimization problem as described under the conditions of IV where the resource stock owner sells the stock, for reasons of optimal portfolio choice and just uses the special consumption value of the exhaustible resource, which is the same as the special extraction value (or special return) of the exhaustible resource. What makes this value of the exhaustible resource special is the opportunity to exchange consumption today for the price of irreversible lower consumption tomorrow.

3. The economy, asset pricing and returns with consumption of the special consumption value of exhaustible resources

Lets define our economy by employing a variation of Lucas (1978)[16] pure exchange model where production is exogenously given by an endowment which follows a Markov process. Since per capita consumption has grown over time, we assume that the growth rate of endowment follows a markov process. It would also be possible to introduce exhaustible resources in a Lucas (1978) framework where endowment follows a Markov process, if we clearly separate the exhaustible resource stock from the Lucas Tree resource stock. Examples for such asset pricing models are given by Breeden (1979)[17], Mehra and Prescott (1985)[18], Duffie and Zame (1989)[19]. There are many extensions in the asset pricing literature with regard to preferences, stochastic processes and diversification as well as alternative frameworks. We could also employ a production-based asset pricing model which concentrates on investments instead of consumption in the framework of Cochrane (1991)[20] and introduce exhaustible resources. Many extensions would also be possible and maybe easy to adjust to this framework. However, the sake of this short abstract is to introduce exhaustible resources, their special characteristics and implications in an asset pricing framework. So this work is concentrated on the most basic applications and relations.

[16] Lucas, R. (1978): "Asset Prices in an Exchange Economy," Econometrica, 46, 1429-1445

[17] Breeden, D. (1979): "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," Journal of Financial Economics, 7, 265-296

[18] Mehra, R., Prescott, E. C. (1985): “The equity premium A Puzzle*”, Journal of Monetary Economics 15 (1985) 145-161. North-Holland

[19] Duffie, D., Zame, W. (1989): “The Consumption-Based Capital Asset Pricing Model,” Econometrica, Vol. 57, No. 6 (November, 1989), 1279-1297

[20] Cochrane, J. H. (1991): Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, The Journal of Finance Vol. 46, No. 1 (Mar., 1991), pp. 209-237

Consider a frictionless economy that has a single representative household who orders its preferences over random consumption paths by:

(7) $E_0 = \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}$, $0 < \beta \leq 1.$

Here c_t is per capita consumption, β ist the subjective time discount factor (according to the rate of pure impatience δ), which describes how impatient households are to consume. $\sum_{t=0}^{\infty}$ is the expectations operator conditional upon information available at time zero, which denotes the present time and U: $R_+ \rightarrow R$ is the increasing, continuously differentiable concave utility function which is of the type of constant relative risk aversion (CRRA) class:

(8) $U(c, \alpha) = \frac{c^{1-\alpha}-1}{1-\alpha}$, $0 < \alpha < \infty$.

The parameter α defines the curvature of the utility function. The CRRA type utility function allows for aggregation and a representative agent formulation that is independent of initial distribution of endowments. Furthermore, it links risk preferences with time preferences, as households who like to smooth consumption over time also like to smooth consumption across various states. The economy has a representative productive unit y_t which produces the consumption good c in period t. There is one equity share that is competitively traded and a claim to the stochastic process $\{y_t\}$. The productive units output is constrained to be less than or equal to y_t, which is the dividend payment in the period t as well. The growth rate in y_t is subject to a Markov chain; that is,

(9) $y_{t+1} = x_{t+1} y_t$.

where $x_{t+1} \in \{\lambda_1, ..., \lambda_n\}$ is the growth rate of production and

(10) $Pr\{x_{t+1} = \lambda_j; x_t = \lambda_i\} = \Phi_{ij}$.

The markov chain is assumed to be ergodic, all λ_i are positive and y_0>0. The realization of the random variable is observed at the beginning of every period, at which time dividend payments are made. All securities are traded ex-dividend. This section follows the approach of Mehra and Prescott (1985)[21]. We will not go into every detail of the stochastic process as for the sake of this abstract it is not necessary. However, in an economy with exhaustible resources we should recognize some details with regard to production functions which could generate $\{y_t\}$ according to a stochastic process with the markov property. Possible production technologies will be scheduled under number 4 and Recursive Competitive Equilibrium with exhaustible resources will be stated.

Let us first have a look on the fundamental relation that prices assets if we have an economy without exhaustible resources which is:

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[21] Mehra, R., Prescott, E. C. (1985): “The equity premium A Puzzle*”, Journal of Monetary Economics 15 (1985) 145-161. North-Holland

(11) $p_t U'(c_t) = \beta E_t\left\{(p_{t+1} + y_{t+1})U'(c_{t+1})\right\}$.

If we apply it on the pricing of the equity share we have:

(12) $1 = \beta E_t\left\{\frac{U'(c_{t+1})}{U'(c_t)} R_{e,t+1}\right\}$,

where,

(13) $R_{e,t+1} = \frac{p_{t+1}+y_{t+1}}{p_t}$, with p_t as the price of the equity share.

For the risk-less one period bond the relevant expression is:

(14) $1 = \beta E_t\left\{\frac{U'(c_{t+1})}{U'(c_t)} R_{f,t+1}\right\}$,

where

(15) $R_{f,t+1} = \frac{1}{z_t}$, with z_t as the price of the risk-free bond.

Now let's introduce the exhaustible resource stock described in section 2 in the economy. Therefore, we focus solely on the special consumption value of the resource stock, which the representative household can realize by consume Cq_t in t, or invest in an equity share of the productive but risky unit y_t , or invest in a risk free bond, or hold the consumption value of the exhaustible resource Cq_t on stock risk free for later periods. Subject to portfolio choice, representative households will only hold portfolios of equity, bonds and the consumption value of the resource stock which are not dominated by other portfolios, otherwise there would be arbitrage on the market. For our economy we focus on the special consumption value (which equals the special extraction value) of the exhaustible resource stock. As the resource owner is assumed to be a price taker, what fits the total assumptions of the model, he will compare the value of keeping the special consumption value of the exhaustible resource on stock, consume it in period t or invest it in the risk free alternative. In fact it makes no sense to keep the special consumption value on stock risk free as long as there is a risk free alternative, which delivers a positive result. Of course we should keep in mind that the resource price in general is correlated to the outcome of the production process y_t and the realization of the markov chain. We have already stated that the representative investor will hold the market portfolio simply by buying resource stocks or shares of the productive units. Therefore the special consumption value or extraction value bears no nondiversifiable risk. A markov process is per definition a process in which the future realization is independent of the past, given the present.

For the exhaustible resource consumption we had the following optimization of the net present value:

(16) $V = \int_0^T u[Cq(t)]e^{-\delta t}dt$.

Where δ is the rate of pure impatience (changed from r to δ to avoid contradictions in the following) which is independent of risk. We have to state clearly that the utility function u is for the marginal rate of intertemporal substitution as there is no risk involved over the consumption path in the "Hotelling economy". Note that we have so far not stated what the value of T will be. It could be the lifetime with respect to Milton Friedman, however it would clearly make sense for the economy as a whole, for future generations but also for reasons of profit maximization that it is infinite.

If we want to introduce the optimal extraction path or exhaustible resource consumption path in an asset pricing model, we should apply the same utility function, where differences in consumption paths over time and states are reflected and where we deal with expectations instead of certain paths. Therefore we transform equation 6 in equation 7 with regard to exhaustible resources:

(17) $$E_0 = \left\{ \sum_{t=0}^{\infty} \beta^t U(Cq_t) \right\},\ 0 < \beta < 1.$$

We should keep in mind that the result of the Hotelling rule, which is the pricing of exhaustible resources, and the result of the pricing of the consumption streams arising from the productive unit should be determined in an equilibrium. We apply the CRRA type utility function in equation 8 for resource consumption. We find that:

(18) $$1 = \beta E_t \left\{ \frac{U'(cq_{t+1})}{U'(cq_t)} R_{q,t+1} \right\},$$

where $R_{q,t+1}$ is the gross rate of return from the exhaustible resource stock.

Now let us compare the gross rate of return from the resource stock with the gross rate of return on equity and bonds. It would not make sense to show the return on the resource stock if there would not be again the special consumption value from the resource stock. But for the reason that this special consumption value of the resource stock is not part of the market portfolio represented by the equity share and its risk free interpretation we should show it separately.

First of all there should be a short comparison between the $R_{q,t+1}$ as groß rate of return on the resource stock and $R_{e,t+1}$ as gross rate of return on the equity share which should be representative for the economies production processes. The return on the equity share consists of stock price increases and the dividend payments, or in other words the price increase and the growth rate of the economies output. As an exhaustible resource stock is a type of equity it should hold that: $R_{q,t+1} = \frac{p_{t+1} + y_{t+1}}{p_t}$. For the reason that the exhaustible resource stock hold in situ or in other words hold on stock is not used in the risky production process, it does not produce a dividend payment in y_{t+1}, if we think in a two period framework of whatever lengths. For this reason the return on the resource stock hold in situ can be rewritten by:

(19) $$R_{q,t+1} = \frac{p_{t+1} + y_{t+1}}{p_t} = \frac{p_{t+1}}{p_t}\ .$$

Although holding a resource stock or the consumption value of the resource stock should be defined as equity, we could more precisely define it as some kind of sustainable hoarding and it makes a lot of sense to compare it to the risk free asset. Again the fundamental relation that prices assets without exhaustible resources is:

(20) $p_t U'(c_t) = \beta E_t\left\{(p_{t+1} + y_{t+1})U'(c_{t+1})\right\}$.

If we apply it on the pricing of the equity share we have:

(21) $1 = \beta E_t\left\{\frac{U'(c_{t+1})}{U'(c_t)} R_{e,t+1}\right\}$,

with p_t as the price of the equity share.

In equilibrium the price of the special consumption value of the resource stock according to a Hotelling path should increase according to at least the risk free rate of return because otherwise it would make sense to extract (or consume) more and invest in the risk free asset.

(22) $\beta E_t\left\{\frac{U'(c_{t+1})}{U'(c_t)} R_{f,t+1}\right\} = \beta E_t\left\{\frac{U'(cq_{t+1})}{U'(cq_t)} R_{q,t+1}\right\}$

Now let's go straight forward and do some unconventional things. First of all we can divide both sides of the equation by the discount factor for impatience β. The new result is obviously:

(23) $1E_t\left\{\frac{U'(c_{t+1})}{U'(c_t)} R_{f,t+1}\right\} = 1E_t\left\{\frac{U'(cq_{t+1})}{U'(cq_t)} R_{q,t+1}\right\}$.

If we take into account exhaustible resources, the rate of pure impatience δ which we can detect on financial markets should be zero in an equilibrium. This sounds unfamiliar but there is an economic reasoning for. If the representative household and therefore the global economy has the opportunity for more current consumption to a distinct grade or rate then they will consume it, no matter what the result for the future consumption may be. This does not mean that any individual household can realize its individual impatience for consumption. But a representative household has per definition shares on every type of exhaustible resources and can realize consumption to the grade of its impatience. We should also have a view on the Hotelling rule of ever increasing prices for exhaustible resources. If the rate of impatience in equilibrium is zero, then the prices for exhaustible resources will not follow an increasing path according to impatience. The extractor will not discount the value of its resource stock with the rate of impatience until an infinite time, as we would assume in an isolated equilibrium, but rather until impatience is satisfied for the economy in every new equilibrium. If we look at economic and financial data and the still and over time more or less constant large share of exhaustible resource consumption like oil, gas, coal or others this seems to be reasonable. If we remember oil price shocks and their impact on global inflation rates and interest rates this gives further evidence.

If we assume a utility function which links risk preferences with time preferences and if such a utility function reflects that human beings want to avoid risk for their consumption opportunities in the future, then we can give a straightforward explanation. The consumption of the “special exhaustible resource value” subject to impatience δ represented in the discount factor β is not independent from the utility function due to the fact that it reduces consumption opportunities in every future state. In other words what is consumed of cq_t in t due to δ has to be adjusted in $\frac{U'(c_{t+1})}{U'(c_t)}$.

Again let's make an example, a person who has a living standard which is so low that he or she is not able to get enough food maybe has a very high rate of impatience but maybe no opportunities to realize it. The person can borrow consumption to a distinct point of debt but this will still not be enough. However, what we should keep in mind if we look at financial markets is that they reflect preferences of individuals only to the grade of their wealth or their wealth opportunities (wealth + opportunities to borrow wealth).[22] Let's assume a very wealthy person may be the owner of a large oil stock who has a very high rate of impatience, which is per definition independent of the wealth and refers more to the individual characteristics. This person will sell a lot of oil, diminish the price of oil on the market and therefore the wealth of may be poorer persons in the economy in the present and then build a large skyscraper. The owner of the oil stock doesn't have to borrow for this as he has the opportunity to consume now, stored in his oil source which he can extract or sell to an investor.

What is left for the valuation of current and past consumption and determines risk free interest rates and therefore the path of exhaustible resource prices is left in equation 20. It's the utility function of CRRA Type with α defining the grade of risk aversion of a representative household as well as expected future consumption growth. If we rearrange the equation we find that the gross risk free rate of return is determined by:

$$(24) \qquad R_{f,t+1} = E_t \left\{ \frac{U'(cq_{t+1})}{U'(cq_t)} \frac{U'(c_t)}{U'(c_{t+1})} R_{q,t+1} \right\}.$$

The expected growth of consumption with respect to exhaustible resources influences the price of a risk free bond z_t and the risk free rate of return. This is of course depending on the utility function. The consumption data and the corresponding growth data can be easily detected on financial markets, if we look at the monetary value of oil, gas, coal, uran as well as metal consumption in e.g. global energy and mining reports.

If we assume in the beginning for reasons of simplicity that α=1 in the utility function in equation 8, then we have by the L'Hôpital's rule that U(c)=ln(c). On theoretical grounds we could refer to the works of Arrow (1971)[23] and concluding that α should be at least close to 1. Application on equation 20 leads to:

$$(25) \qquad E_t \left\{ \frac{c_t}{c_{t+1}} R_{f,t+1} \right\} = E_t \left\{ \frac{cq_t}{cq_{t+1}} R_{q,t+1} \right\}.$$

Rearrange the equation after the risk free rate of return yields to:

$$(26) \qquad E_t \left\{ R_{f,t+1} \right\} = E_t \left\{ \frac{c_{t+1}}{c_t} \frac{cq_t}{cq_{t+1}} R_{q,t+1} \right\}.$$

[22] Sharpe, W. (2011): "Investors and Markets: Portfolio Choices, Asset Prices, and Investment Advice", Princeton University Press

[23] Arrow, K.J., 1971: "Essays in the theory of risk-bearing" (North-Holland, Amsterdam)

Rearrange the equation after the rate of return on exhaustible resource assets yields to:

(27) $$E_t\left\{R_{q,t+1}\right\} = E_t\left\{\frac{c_t}{c_{t+1}}\frac{cq_{t+1}}{cq_t}R_{f,t+1}\right\}.$$

A positive expected growth in aggregate consumption would increase the risk free rate of return and positive expected growth in exhaustible resource consumption decreases the risk free rate of return. Now let us state a new equilibrium based on risk neutral preferences simply by dividing both sides of the equation by the expected gross risk free rate of return:

(28) $$1 = \frac{E_t\left\{\frac{c_{t+1}}{c_t}\frac{cq_t}{cq_{t+1}}R_{q,t+1}\right\}}{E_t\left\{R_{f,t+1}\right\}}$$

What we find in equation 23, 24 and 25 is, that it is the relation between the expected growth factor of aggregated consumption and the expected growth factor of consumption subject to exhaustible resources which determines the risk free rate of return and the rate of return on holding the consumption value of exhaustible resources, which is in fact a time value. If aggregated consumption and exhaustible resource consumption grow with the same rate, the return on holding the consumption value of exhaustible resources and holding the risk free asset will be equal. This condition is based solely on portfolio choice considerations, if there is a difference then investors will shift their investments from exhaustible resources to the risk free asset and vice versa. What determines the return on the risk free asset and aggregated consumption is the realization of the markov process variable $\left\{y_t\right\}$ which should be interpreted as an exogenous productivity variable of the general production process. The owners of exhaustible resources adjust to every new realization of $\left\{y_t\right\}$ based on arbitrage considerations.

How can we interpret this under considerations of a sustainable treatment of exhaustible resources and with regard to the climate debate. First of all this result can be interpreted in the form that investors, resource owners or consumers adjust to changes in the relation between aggregated consumption growth and exhaustible resource consumption growth in every new equilibrium. A sustainable path under arbitrage considerations would be a path where aggregated consumption and exhaustible resource consumption grow with the same rate. If there is an abbreviation from this “sustainable” path, the representative household or the economy as a whole will adjust under arbitrage considerations. If the consumption of e.g. oil increases with a much higher ratio as the general consumption, the risk free return decreases as the representative household has a higher risk for future consumption and prefers to invest more in the risk free asset. Equation 22 is a stable equilibrium interrupted constantly by shocks due to the realization of the Markov-Process $\left\{y_t\right\}$.

Looking at the development of resource extraction over time, measured in physical units as well as in monetary values, it is increasing instead of decreasing and prices do not follow an increasing path. The graphics are based on the analysis of 2013 and 2014 but it is straightforward to go ahead with the data and recognize that adjustments follow a shock process, where even negative e.g. oil prices have been realized meanwhile. Overall there is a growing trend beside or even further through technological changes like digitalization, which some thought would reduce the share of resource extraction. In this regard we could also think about the share of exhaustible resources on overall gross domestic product as a crisis indicator. To be precise and follow the analysis, we should think that consumption growth subject to exhaustible resources, which exceeds the economy's overall consumption growth, leads sooner or later to adjustments in the form of a crisis.

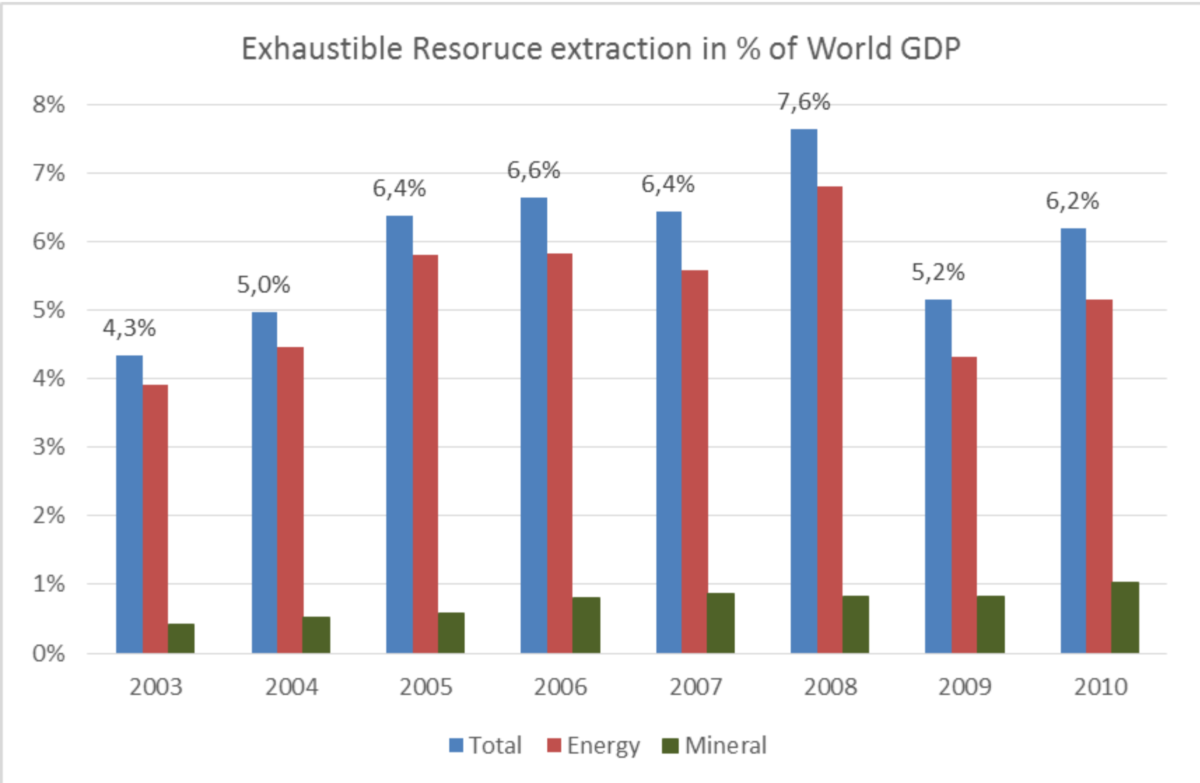

Figure 2: Exhaustible resource extraction in % of global gross domestic product (USD notation)
Source: World Bank, U.S. Geological Survey

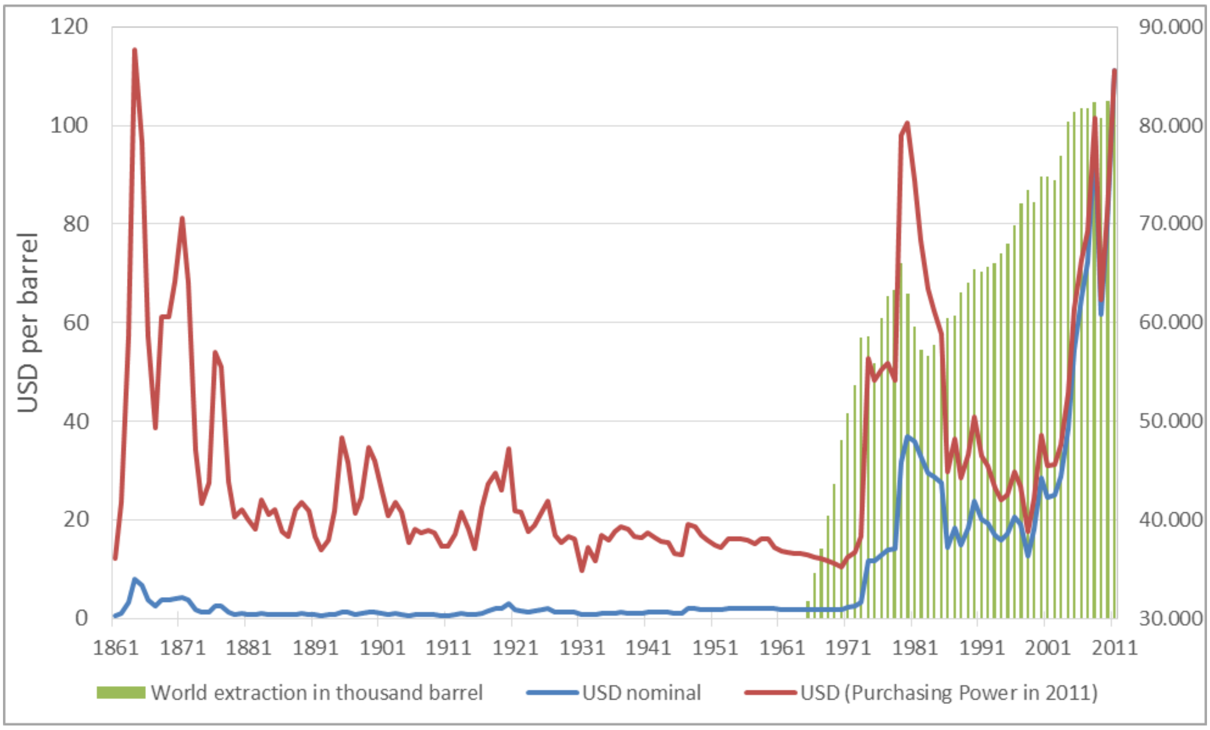

Figure 3: Physical and monetary extraction of crude oil (USD notation in Million)
Source: BP Statistical Review of World Energy 2012, U.S. Geological Survey

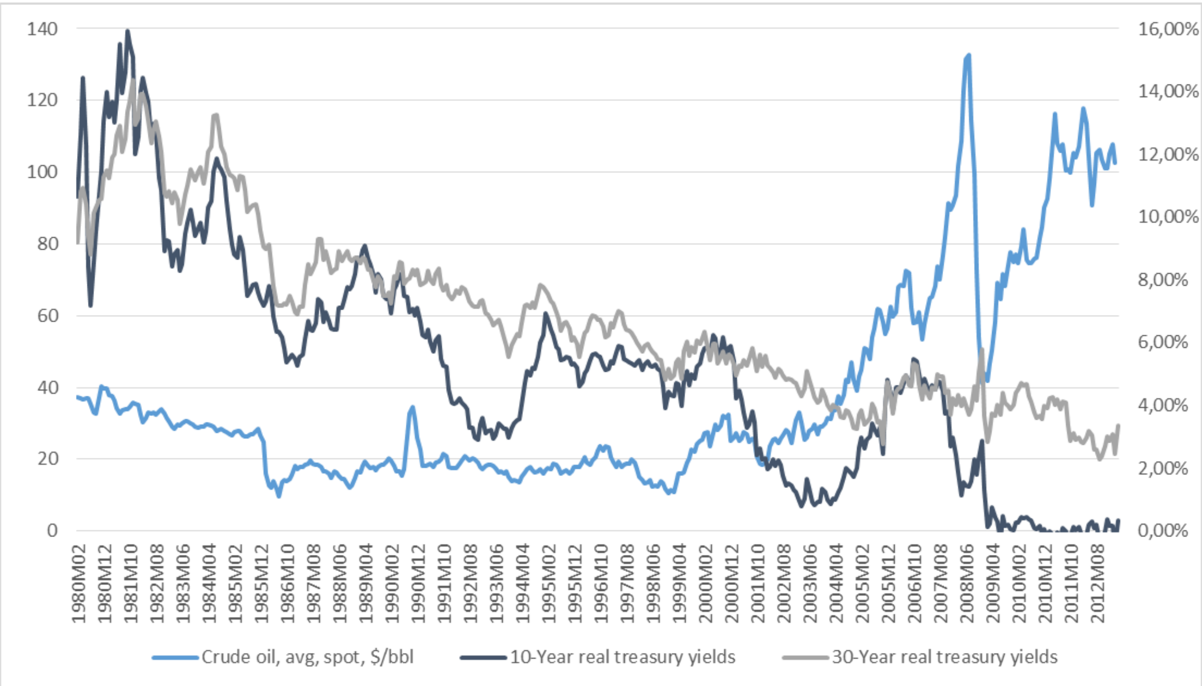

Figure 4: Crude oil price development and interest rates
Source: World Bank, Bureau of Labor Statistics

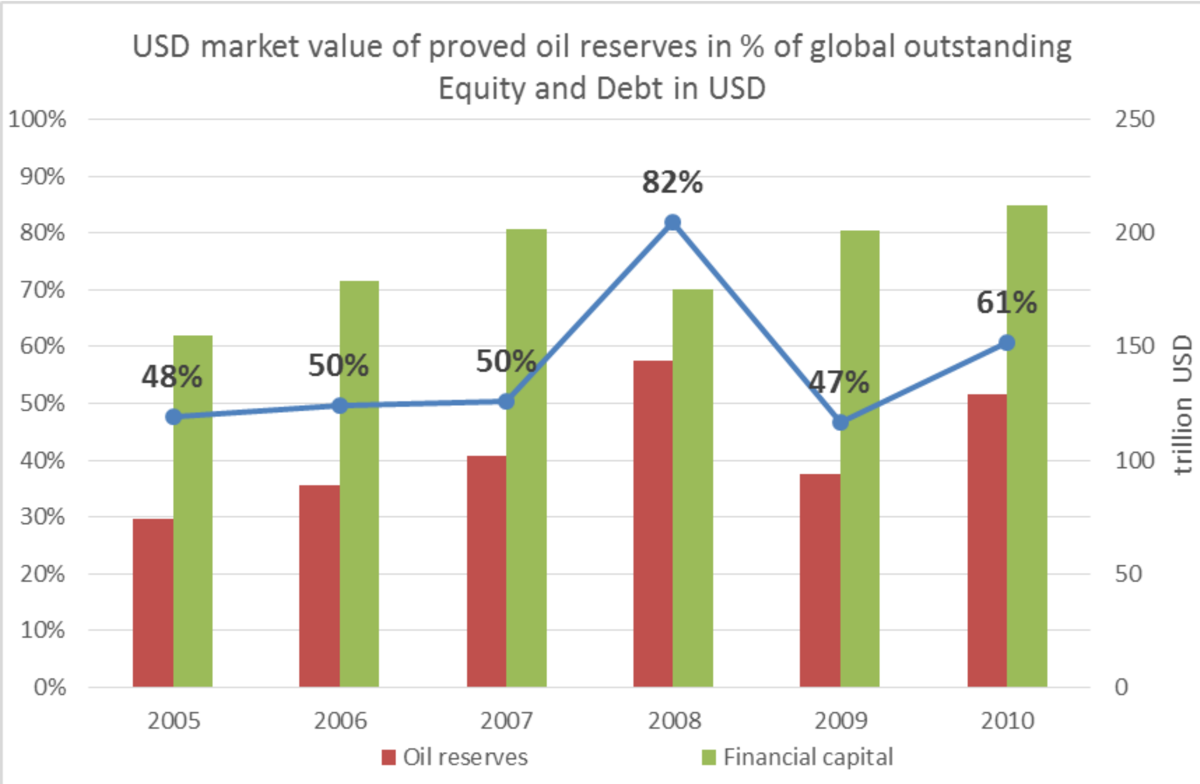

Figure 5: Proved oil reserves and there relation to outstanding financial capital
Source: BP Statistical Review of World Energy 2012, McKinsey & BIS (Mapping global capital markets 2011)

4. Production technology, functional stochastic processes and a Recursive Competitive Equilibrium

Let us have a look at the process, which generates the dividend of a productive unit or in an aggregated model the gross domestic product of the economy.

(29) $y_t = f_t(L_t, K_t, Q_t, \lambda)$

Therefore, we define a production function that links labor defined by L_t, capital defined by K_t (capital is commonly referred to as aggregated labor and exhaustible resources [e.g. machines out of metal] or aggregated human capital [aggregated knowhow, plans, patents, or abilities of human workers]. In addition we define exhaustible resources as a production input (we have to differentiate between the consumption subject to Q (cq_t) and the production input variable Q). Productivity is defined as λ and follows a stochastic process if we assume productivity independent from the other inputs. It is also a term which defines state variables or exogenous shocks which follow a stochastic process like a pandemie, a new ice age or an asteroid that hits the earth.

If we assume an exogenous productivity variable we could imagine e.g. a Cobb-Douglas type production function of the following form:

(30) $y_t = f_t(L_t, K_t, Q_t, \lambda) = f_t(L_t^{1-a-b} K_t^a Q_t^b \lambda_t)$.

This production function is scale invariant and tells us at least that there is some type of optimal relation in which we should use L, K and Q in an economic or sustainable way. Note that it is of course possible that this production function and therefore the optimal parameters a, b and 1-a-b change over time. Further we denote λ_t as our productivity (usually total factor productivity) which we define to be the only stochastic part of the production function, which is given exogenously. This is a typical macroeconomic production function with the extension of the exhaustible resources used in Solow (1974), Stieglitz (1974), Gaitan, Tol, Yetkiner (2006). If we look at this production function we can define productive inputs of labor, capital and exhaustible resources for which a firm optimizes its production plan. If we look at the marginal product of capital, which should in equilibrium equate to the real rate of interest and the marginal product of the resource we could find supply side solutions. These solutions will not take into account the utility functions of people, which should represent their basic needs, especially for investments, which are monetaire if someone looks behind the curtain. We can also define some obvious facts from this. Based on an economic and a sustainable perspective a firm or an economy should employ production factors, which are available any way (which is labor and already produced capital) and try to save the production factor which may be consumed within production, which is the exhaustible resource. We further have to think about how capital is aggregated and how capital like machines are depreciated by use and time. As capital depreciates over time it has to be employed for productive use and if it depreciates by extensive use the firm should assume that it does not employ it extensively. There is of course a mathematical optimum for all of these points which the firm in theory can calculate.

However, capital is aggregated by investments in the capital stock and not by saving. If a representative firm saves its output, which is the consumption good, it will obviously not have a larger machine stock and if an individual saves the consumption good it will also not have a larger capital or machine stock. If we assume a representative firm or production technology, then the firm has to find an optimal decision between the production of K or C. In reality there are different types of firms with different technologies of which some produce K (e.g. assembly machines, mechanical engineering) and some produce the consumption goods and some produce both. Of course we can further complicate this by taking into account that some firms or institutions produce Human capital (brain capital) e.g. by education then we could also imagine this as aggregation of K. However, if we keep in mind the term “learning by doing” then we should take into account that K aggregates also by working on the job and not only by education.

If we look at our Cobb-Douglas-type production function or at some existing real relations, we find some very interesting and logical facts with regard to sustainability. Within the production function, which was developed by Charles Wiggins Cobb and Paul Howard Douglas because it fits the aggregated economic data very well, we can substitute for example labor L for exhaustible resources Q. In reality we could imagine working less and therefore consuming more oil, gas, uran or other resources to produce energy or run computers and machines and then we have the same output in period t to the price of a non-optimal higher use of the exhaustible resources. If we take into account endogenous growth theory, what means for example that people work together or alone on a topic and then develop ideas while working on the problem and if this drives productivity up beside the of course existing random process variable λ_t, we should think about the consequences for our society in the mid or long term if we work less.[24] There are undeniable firms or institutions which are teaching people how to work and educate in general and if we think about such institutions not working, then we could also think about how this diminishes our capital stock or the productivity of our society. But then we can use more oil or energy to compensate for this lost productivity or capital stock in the future.

The problem we find if we apply this production function is that either the term b or Q itself could become zero and therefore the total production would become zero. This is maybe as realistic as could be imagined, that we always need some exhaustible resource in the production but there is another problem with this type of production function. As we have stated above, resource Q is distributed randomly above the earth, can not be reproduced or safed by humans or to be more precise its economic properties are not reproducible with existing technologies. It follows that the amount of resource Stocks of Q is definitely unknown but over a long time there are additional explorations and also opportunities to use Q or maybe substitute any kind of exhaustible resource with another. It seems that it is not only the overall stock of Q which is unknown but furthermore the exploration and the useability of the resource is random. And if we at least refer to the meteorite that hits the earth just by chance, we should keep in mind that this could increase the stock of Q. Therefore, if we separate between the time value and behavior toward risk at the consumption side, we can separate between deterministic and stochastic parts of production or to be more precise real investment opportunities.

$$(31) \quad y_t = f_t(L_t, K_t, Q_t, \lambda_t) = f_t(L_t^{1-a} K_t^{a})\lambda_t(Q, T)$$

In equation 31 we have a type of production function that uses capital and labor in the deterministic form where capital can be accumulated and where the resource stock of Q as well as the overall productivity follows a stochastic process. In other words the exhaustible resource is a part of productivity itself or a type of superior real investment opportunity. This means exactly, that wherever in time we are looking at exhaustible resources, their stock as well as their productivity is a realization of the stochastic process, subject to new explorations. Following this, the realization of the stochastic process at any time t is independent of past decisions and optimal extraction paths are realized to the information given in the present. The disadvantage of Equation 31 is, that it does not reflect the characteristic special return or scarcity rent of the exhaustible resource. Furthermore it is not a realistic assumption, that people with rational expectations have no knowledge about the

[24] Romer (1990): Endogenous Technological Change. In: Journal of Political Economy. Band 98, Oktober 1990

stocks of ore, oil or whatever resources at al. Strong analysis with a lot of affordance is done at every point of time from oil and mining companies as well as geologists. The knowledge about the resource stocks can be assumed to improve over time.

If we recognize the Hotelling rule as a valid decision problem, although including risk, we have in any case a circular reference if we look on the demand side. The resource owner does not generate utility from the consumption of the resource, especially if we recognize concentrated resource stocks subject to the nature of their explorations and simple natural conditions. But these problems can be fixed within e.g. a recursive competitive equilibrium, which fits to the basic approach under section 3 and shows a valid closed form solution.

Therefore we follow the approach of Mehra (2005): “Recursive Competitive Equilibrium”[25] with an adjustment for exhaustible resources. Usually production functions are assumed to produce either the single consumption good C_t and the capital stock K_t. The consumption good is commonly defined as the numeraire good of the economy, which is the source of value. Within the Recursive Competitive Equilibrium time-invariant equilibrium decision rules are applied, which specify current actions as function of a limited number of “state variables”, which fully summarize the effects of past decisions and current information available. Knowledge of these state variables provides the economic agents with a full description of the economy's current state. The actions of agents following time invariant decision rules together with exogenous uncertainty defined by λ_t, which is the realization of a stationary Markov process with a bounded ergodic set, defines a pareto optimal market equilibrium. The pareto optimum will prove that the utility functions of individuals aggregate to the utility function of a representative agent.

Starting point is the central planning stochastic growth paradigm which uses the utility function of a representative consumer denoted in in Equation 7:

(32) $$w(k_o, \lambda_0) = maxE\left\{\sum_{t=0}^{\infty} \beta^t U(C_t)\right\}$$

subject to: $c_t + k_t \leq f(k_t, l_t)\lambda_t,\ \ \lambda_0, k_0$ given, $l_t = 1 \forall t$

We only define the new notations, so k_t denotes capital available for production in period t and l_t denotes period t labor supply, where it is assumed that it is supplied in-elastically by the consumer-investor at $l_t = 1$, for all t. This assumption is sufficient for the proof of this abstract, however it would be interesting to apply more sufficient solutions with regard to the topic of exhaustible resources and employment. The production function $f(k_t, l_t)\lambda_t$ represents the technology in period t, which is shocked by the bounded stationary stochastic factor. E again denotes the expectations operator and the central planner is assumed to have rational expectations and uses all available information to rationally anticipate future variables. This means he knows the conditional distribution of future technology shocks

[25] Mehra, R. (2005): “Recursive Competitive Equilibrium”, prepared for The New Palgrave Dictionary of Economics, 2nd edition

$F(\lambda_{t+1}, \lambda_t)$. For the equilibrium proof we assume that in equation 8 U(c)=ln(c) and we assume a Cobb-Douglas technology like described in equation 30 $f(k_t, l_t) = k^a l^{1-a}$. We still follow Mehra (2005) and assume that $a < 1$, $\beta < 1$ and that capital fully depreciates each period. This assumption is not unrealistic for equilibrium conditions due to the fact that the household supplies labor and capital competitively to firms in every period and consumes the consumption good and invests (safes) in the capital stock. Based on this conditions a closed form solution to the planning problem exists:

(33) $c_t = (1 - a\beta)k_t^a\lambda_t$, and $k_{t+1} = i_t = \alpha\beta k_t^a\lambda_t$

Investment in the capital stock is defined as i_t and is consumption held over for production in period $t + 1$.

In equation 33 consumption is the proportional part of labor in period t on the overall production, where the proportional part of capital on the production is reduced by the discount factor for impatience. The proportional part of labor on the overall production are wages and impatience for consumption leads to higher consumption. The overall production is of course subject to capital income, which is the marginal product of capital which equals the real rate of return on capital in equilibrium. Wages and capital incomes are of course both subject to the stochastic productivity level.

Investment: As capital fully depreciates in period t the capital stock in period $t + 1$ is equal to investments in period t, which are equal to the proportion of capital on the total production in period t multiplied with the discount factor for impatience for consumption. This is of course although subject to the stochastic productivity in period t.

This allocation is a pareto optimum.

Discussion of the stochastic process:

We should recognize that the ergodic bounded stationary stochastic factor follows a markov process with a zero-mean. This process does not replicate the nonstationar properties of equity share prices but it replicates the price process for commodities or to be more precise exhaustible resources like oil, gas or metals.

However, the nonstationary properties are reached by the discount factor for impatience which leads to a reduced saving (saving = investment in equilibrium) in period t and therefore scarcity of capital in period $t + 1$. Therefore a positive price trend for accumulated capital (in this approach with 100% depreciation) is generated, which reflects a return.

Exhaustible Resource:

We have a difference to the production function in equation 31, we are missing the exhaustible resource. If we want to introduce exhaustible resources in the recursive competitive equilibrium based on time invariant decision rules, we have to do this now. The Hotelling rule is a time invariant decision rule and tells us that exhaustible resource owners supply their resources subject to the conditions already described in equation 6, 17, 18 and 19:

(34) $V = \int_0^T u[Cq(t)]e^{-\delta t}dt \ -> R_{Q,t+1} = \frac{p_{t+1}+y_{t+1}}{p_t} = \frac{p_{t+1}}{p_t} = \frac{p_{t+1}}{p_t} - 1 = \delta$

Exhaustible resources are assumed to be supplied competitively to the firm, fully depreciate within one period and have to be supplied in the following period to the firm. Within this framework this is all equal to accumulated capital. However, we still have the term of the scarcity rent for the exhaustible resource due to the fact that they are a reserve for higher production in period t. We can look at this point either as a substitute for accumulated capital, e.g. additional crude oil or natural gas for energy generation instead of a wind energy generator or a solar module so equation 30 would fit.

Note that we are still on the level of a centralized planner, however the Hotelling rule is a rule which connects the decisions of an individual with the decisions of the overall economy. In other words the Hotelling rule is a rule where the owner of the exhaustible resources has to adjust to the rate of impatience of the market, which we will denote in the following witt δ_M and the corresponding discount factor of the market β_M^t and not to its own impatience. In other words resource owners who maximize their individual return (individual utility) adjust to the rate of impatience of the economy, which is determined by individual decision makers subject to their wealth distribution. This shall outline the central role of natural resources and the Hotelling rule for economics.

Let us obtain a closed form solution now and then show that the Recursive Competitive Equilibrium in a decentralized economy leads to the centralized version:

(35) $f_t(L_t^a K_t^b Q_t^c)\lambda_t = f_t(L_t^{1-a-b} K_t^a Q_t^b)\lambda_t$

subject to $V(Q_0, \lambda_0) \ = maxE\left\{\sum_{t=0}^{\infty} \beta_M^t U(Cq_t)\right\}$

where $Cq_t = p_{qt}Q_t$, $u[Cq(t)]$=ln($p_{qt}Q_t$) and $\beta_M^t \ = (\frac{1}{(1+\delta_M)^\wedge t}) \equiv e^{-\delta_M t}$

So far the exhaustible resource extraction is the only sector, which we can describe on the level of a central planner due to the direct link to the market utility function and discount rate.

Supply of the exhaustible resource:

First of all we should recognize that the Hotelling rule tells us the amount of the resource Q which is supplied to production or to a profit maximizing representative firm. The answer to the question about the quantity is in fact the rate of impatience and not an increasing price over time. It is a surplus income subject to scarcity but to be more precise subject to immediate ability to increase consumption (production). The Cobb-Douglas production function is:

(36) $y_t = f_t(L_t, K_t, Q_t, \lambda) = f_t(L_t^{1-a-b} K_t^a Q_t^b \lambda_t) = f_t(L_t^{1-a-\delta_M} K_t^a Q_t^{\delta_M} \lambda_t)$,

where $b = \delta_M$.

Within the Cobb-Douglas production function output elasticities are constants determined by available technology and given over a long time Horizont. First we interpret the more common output elasticity of capital a. It tells us how much the overall production increases for an infinitesimal increase of capital K and furthermore it tells us the capital income ratio on overall production. Now let's interpret the output elasticity of the exhaustible resource Q which is defined as b:

- ❖ how much does overall production increase for an infinitesimal increase of the resource Q -> equals to equation 34
- ❖ The exhaustible resource income ratio on overall production determined by available technology (subject to λ_t)

In conclusion the Hotelling rule directly connects the supply of exhaustible resources from a single producer with the economy's preferences and it connects production with utility and therefore consumption. This is a time invariant rule which connects intertemporal decisions with the general production frontier between different alternatives. In other words, for whatever period of time or point of time with given production technology (subject to λ_t) and given exhaustible resources stock, the optimal quantity supply is determined by the impatience for current consumption subject to the preferences of the economy.

(37) $MPQ = \frac{\partial f(f_t(L_t^{1-a-b} K_t^a Q_t^b)\lambda_t}{\partial Q} = bK_t^a Q_t^{b-1} \lambda_t$

$for\ Q = 1\ and\ L = 1 \quad MPQ = b\, K_t^a \lambda_t = \delta_M K_t^a \lambda_t,$

The exhaustible resource will be supplied subject to the rate of impatience for consumption. The exhaustible resource owner will supply a share which is consistent with the rate of impatience multiplied with the gross domestic product, for inelastic labor supply at 1. The exhaustible resource owner earns his marginal product, which is a surplus income of δ_M . The owner sells to the production cost plus a premium for immediate ability of consumption at δ_M. Subject to impatience, exhaustible resources are supplied. This supply increases production and therefore consumption. The investment in the accumulated capital stock is reduced by $\beta_M \left(\frac{1}{(1+\delta_M)}\right)$. This means exhaustible resources are supplied as a substitute for not available accumulated capital at time t. The premium on exhaustible resource prices guarantees additional investment in the accumulated capital stock as a substitute.

Therefore, we can adjust the pareto optimum closed form solution of the central planner only in the interpretation:

(38) $c_t = (1 - a\beta)k_t^a \lambda_t = (1 - a\left(\frac{1}{(1+\delta_M)}\right))k_t^a \lambda_t,$

and $k_{t+1} = i_t = a\beta k_t^a \lambda_t = \alpha\beta_M k_t^a \lambda_t = a\left(\frac{1}{(1+\delta_M)}\right)k_t^a \lambda_t$

Following this analysis of the characteristics of the exhaustible resource there is no analytical reasoning why we should not treat exhaustible resources as part of accumulated capital, as the special time value of them. Following this, the author refers directly to Mehra (2005): "Recursive Competitive Equilibrium page 8ff" to prove that a decentralized economy will find the pareto optimal equilibrium according to 38. The author further refers to e.g. Donaldson and Mehra (1983), Mehra and Prescott (1985) and Mehra (1988) for an application on risk premium as well as to an application with non-stationary consumption. It is noteworthy, that the supply of exhaustible resources with a premium for impatience for consumption in the price is a continuous process. It further seems to be the first process of human economic interaction if we remember the hunter-gatherer age where an even larger amount of resources were exhaustible. Within the age of farming the use of exhaustible resources may have decreased but within industrialization and digitalization it seems to have increased again. While of course the commodities behind the notation Q have changed due to technological progress.

In fact equation 37 tells us that the discount factor according to impatience is itself the scarcity rent for exhaustible resource extractors. This property is also valid in general equilibrium models as well as in e.g. production based asset pricing. It is also valid in an endowment economy with an additional exhaustible resource stock, which allows for additional current consumption (endowment). We should recognize that within the term $(1 - a\beta)k_t^a\lambda_t$, β is the reason why exhaustible resource extractors supply the resource to the production process to generate additional consumption now, to the price of lower consumption in the future.

Observation of the rate of impatience and thoughts about sustainability:
The author has stated above, that impatience can not be measured on financial markets. Well the unobserved true rate of impatience of the global economy can be measured as suggested in equation 38 by the ratio of the monetary exhaustible resource consumption value divided by the global gross domestic product (monetary value). We find these ratios in Figure number 2. Based on thoughts of sustainability impatience for consumption lays in the long term average around 5% to 6%. But we should recognize that exhaustible resources bear this as a time invariant scarcity rent or premium which is a constant. This makes their use and therefore their depletion expensive for a profit maximizing firm and this is a garant for sustainability based on economic rules. The impatience for consumption limits unsustainable growth and therefore the depletion of exhaustible resources for economies which grow without technological progress.

Abbreviations from the Hotelling rule: It is worth mentioning that there are existing treats for exhaustible resource owners who could lead to the decision to adjust to their own time preference. This would be not an economic and rational decision but it could be the decision of choice e.g. for political motives. As natural resources like oil or gas are highly aggregated in countries of distinct regions in the world for natural reasons but although for reasons of current depletion (e.g. the United States of America had not to long time ago also large oil stocks with comparable profitability of Oil reserves e.g. to Iran or Saudi Arabia, but these are depleted now while less profitable are available due to new technologies). The aggregation and the political control (state control) over natural resources could afford to use natural resources subject to individual preferences of countries or their government. Of course a state could have political reasons for not selling his resource stocks to firms of foreign countries. However the foundation and extraction decisions of Organizations like OPEC shows that governments have understood and follow economic reasoning, beside geopolitical aspects.

5. Conclusion

This abstract is just a working ground for the introduction of exhaustible resources in Asset Pricing and their influence and interaction with impatience, behavior toward risk and intertemporal substitution of wealth. It should build a bridge between general equilibrium models, optimal extraction path and stochastic asset pricing models. If we look at existing asset pricing puzzles like the equity premium and the risk-free rate puzzle, which are associated with consumption based asset pricing on the one hand and on the “Hotelling rule puzzle” of not ever increasing exhaustible resource prices on the other hand, this abstract could deliver a plausible solution for both.

The production technology and the Recursive Competitive Equilibrium is a simplified model for a closed form solution to prove that a pareto optimal equilibrium exists.

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